C. Rujescu, et al. Scientifical Researches. Agroalimentary Processes and Technologies, Volume XI, No. 2 (2005), 499-502

THE GRADIENT METHOD IN THE DETERMINATION OF THE EXTREMES OF A PRODUCTION FUNCTIONS

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Abstract

The determination of the extremes of the function with more real variables often implies difficulties because of the relatively complicated calculations that may appear if we try to solve these problems by using the classical methods of the mathematic analyses. These very aspects and the fact that many processes from the agro-alimentary economics have as mathematic model the functions with more variables ask for and prove the utility of some studies regarding the creation of algorithms for solving the above mentioned problems Therefore proposes this work to adopt a known method in the speciality literature, that is the gradient method, in order to shape a new calculation method in the ago-alimentary economy.

Keywords: *The gradient method, the extremes of the functions with more variables, agro-alimentary economy*

General Notions

The dependency between the production and the production factors may be expressed by a real function with n variables:

$$\mathrm{f} : \mathrm{A} \subset \mathbf{R}^{\mathbf{n}} \rightarrow \mathbf{R}.$$

with

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \ldots, \frac{\partial f}{\partial x_n} \right)$$

as the expression of its gradient.

It is known that the sign and the mass of the gradient's components indicate the way function **f** varies. Starting from these it can constitute an algorithm which starts from an arbitrary point from the definition domain of the function and according to the value of the partial derivates of the function in that point it can be "improve" the values of its components meaning the shifting towards a point of extreme (a point in which the partial derivates annul themselves) (Vaduva, 1974).

It must notice from the beginning that in a general case (that is for a function which has more local extreme points) the algorithm may lead us to values very distant from the absolute extremes. Although, as shown in the above mentioned paragraphs, in the case of the production functions which shape the processes in the agriculture, these situation does not appear: these functions have only one extreme point and we approximately know the interval in which the extreme values are to be found (Cret, 2002).

It uses most often functions having the following form:

$$Q = a_1 + a_2x_2 + a_3x_1^{\ 2} + a_4x_2 + a_5x_2^{\ 2} + a_6x_1x_2$$

where ***Q*** represents the production and $\boldsymbol{x_1}$ and $\boldsymbol{x_2}$ the quantities which correspond to each production factor (Otiman, 2002).

This fact is very useful because the starting point is relative close to the theoretical extreme value, so that the number of iterations decreases very much.

Application

Let's take into consideration a function of production of the form presented above:

$$Q = 3600 + 160N - N^2 + 200P - P^2$$

Annulling the derivates are obtain:

$$\begin{cases} \dfrac{\partial Q}{\partial N} = -2N + 160 \\ \dfrac{\partial Q}{\partial P} = -2P + 200 \end{cases}$$

$$\begin{cases} -2N + 160 = 0 \\ -2P + 200 = 0 \end{cases}$$

Solving this system, $N = 80, P = 100$ are obtained.

If the gradient method is applied, starting from the values

$N=50$

$P=50$

we have:

a) $\nabla f = (60,100)$.

It has to be noticed that both values are positive, so that the function is growing in comparison with each component. So, in order to get closer to the maximum we maintain a constant component and raise the other with the value "t". We have:

N = 50, P = 50 + t.

In order to determine the optimum value of **t**, it will optimise the function:

$F(t) = f(50, 50+t) = -t^2 + 100t + 16600$

The derivate of this function is:

$F'(t) = -2t + 100$

Annulling the derivate t = 50 is obtains.

b) N = 50

P = 100

and

$\nabla f = (60,0)$.

So:

N = 50 + t

P = 100

and

$F(t) = f(50+t, 100) = -t^2 + 60t + 19100$

$F'(t) = -2t + 60$ it results t = 30 after annulling the derivate.

c) N = 80

$$P = 100$$

and

$$\nabla f = (0,0)\ .$$

It is to be noticed that the solution to the problem is obtained by doing three iterations.

Conclusions

It is to be noticed the convergence speed of the algorithm and in the same time the possibility of implementing of an informatics program having at the basis such an algorithm. To be noticed is also the creation of links between different domains of science by adopting of some intensively studied but insufficiently applied methods, which have, as we can see, a very important utility.

References

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