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SOME ASPECTS REGARDING THE MAXIMIZATION OF PROFIT

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Abstract

This work presents some aspects regarding the interpretation of a mathematic model attached to a permanently actual subject: the maximization of profit (in the agro-alimentary economy). We fist present the theoretical part from the economic – mathematical point of view as it appears in the specialized literature, using as model the functions with more variables and the unconditioned extremes. The model will be then geometrically analyzed and we will create the general frame in order to build an algorithm for solving it.

Keywords: *functions extremes, profit maximization*

The Profit Maximization

Be **Q** the productivity and **p** the selling price of an agro-alimentary product. Be x_i, i = 1,…, n, the used quantities of the production factors and $\mathbf{p_{xi}}$ their price. Let's suppose the producer has the possibility of operating not only on the production, but also on the costs and then implicitly on the difference between the income and the costs, that is on the profit (Otiman, 2002). If the function which expresses the connection between the production and the factors of production is known:

$$Q = F(x_1, x_2, \ldots, x_n)$$

then it has to optimize that function

$$P = p \cdot F(x_1, x_2, \ldots, x_n) - \left(p_{x_1} x_1 + p_{x_2} x_2 + \ldots + p_{x_n} x_n\right)$$

which represents the difference between the incomes and the costs.

Between the variables x_1, x_2,…, x_n are no restrictions, so it can apply the algorithm for determining the extremes of the real functions with more variables (Cret, 2000). So:

$$\frac{\partial P}{\partial x_i} = p\frac{\partial F}{\partial x_i} - p_{x_i} = 0$$

and as a consequance

$$p_{x_i} = p \cdot \frac{\partial F}{\partial x_i},\ i = 1,...,n .$$

The price for each factor is equal to the marginal productivity $\frac{\partial F}{\partial x_i}$, in the value expression (Mansfield, 1975).

Algorithms for the Determination of Non-conditioned Extremes

Let's notice that the problem has reduced to the maximization of the profit function a function with *n* real variables:

$$P(x_1, x_2,..., x_n) = p \cdot F(x_1, x_2,..., x_n) - (p_{x_1} x_1 + p_{x_2} x_2 + ... + p_{x_n} x_n)$$

The degree of difficulty of the problem grows for some expressions of the function F as well. That is why it is necessary to study some algorithms that should be used in a computer program in order to obtain the wanted results (Intriligator, 1971).

It has to remind the definition of the maximum points of the real functions with more real variables: the points,

$$(x_1{}^0, x_2{}^0,..., x_n{}^0) \in \mathbf{R^n}$$

so that for any $(x_1, x_2,..., x_n) \in \mathbf{R^n}$ there are:

$$f(x_1{}^0, x_2{}^0,..., x_n{}^0) \geq f(x_1, x_2,..., x_n)$$

for the maximum, and:

$$f(x_1{}^0, x_2{}^0,..., x_n{}^0) \leq f(x_1, x_2,..., x_n)$$

for the minimum.

In the general case of a function with more real variables, a unguided search of the extreme points along the whole variation domain of the factors of production leads to a very large number of iterations, which implies a long time needed in order to solve the problem. Exactly these facts call for the use of some searching strategies.

Using the information that from the practice it generally knows the limits imposed by using a factor of production can also restrict the search (Otiman, 2002).

Because of graphic reasons, the geometrical study could be realized for a function with two variables, z: $A \subset \mathbf{R}^2 \to \mathbf{R}$, $z = y(x,y)$.

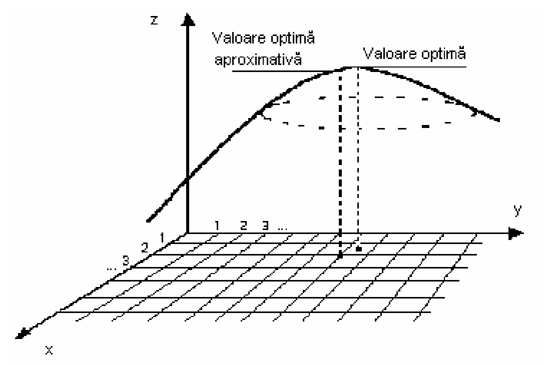

Fig. 1. The division of the Oxy plane

If it is considered on the axes Ox and Oy some divisions consisting in "n" units, the plane Oxy can be divided in "n^2" squares all having the surface equal with the unit (figure 1). It is therefore clear that an unguided search is not at all recommended. In order to shorten the time necessary in order to solve the problem it can do the search for divisions of the intervals larger than the unit. Once settled an approximate position of the optimal points, the search will be done around these points by using a finer division (Rujescu, 2005).

So, if is chosen an equidistant division having the norm equal with "p" units and the maximum returned value ("the approximate optimal value 1") corresponds to the point $M_1(i, j)$, then we will do the search for values belonging to the square $M_2M_3M_4M_5$, where: $M_2(i-p, j-p)$, $M_3(i-p, j+p)$, $M_4(i+p, j+p)$, $M_2(i+p, j-p)$ (figure 2).

Application: If it is given the profit function:

$$P = 245 + 19.23N - 0.0612N^2 + 9.13P - 0.051P^2 + 0.0104NP$$

results:

$$\begin{cases} -0.1224\text{N} + 0.0104\text{P} = -19.23 \\ 0.0104\text{N} - 0.102\text{P} = -9.13 \end{cases}$$

and so

N = 166.152, P = 106.450

If is solved the algorithm suggested in order to determine the extremes, it will obtain the whole values:

N = 166, P = 106.

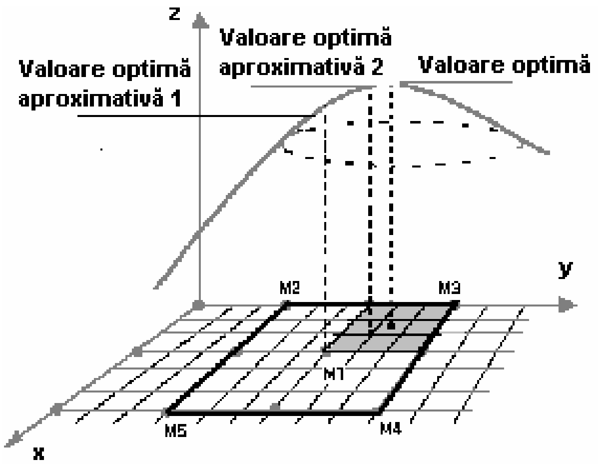

Fig. 2. Finer division around the opimal value

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