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ASPECTS REGARDING THE LAGRANGE MULTIPLICATION METHOD IN MATHEMATICS PROGRAMMING AND IN ECONOMICS

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Abstract

The Lagrange multiplication method is well known in the ago-alimentary economical practice, more exactly in the situation in which the selected model needs the determination of the extremes of the real functions with more variables on which restrictions are to be imposed. The work represents a geometrical study of the method and is a support for its algorithmisation.

Keywords: *functions extremes, Lagrange method*

The Description of the Lagrange Multiplicators Method

Let's consider a real function with many real variables: $f : A \subset \mathbf{R}^n \to \mathbf{R},\ f = f(x_1, x_2, ..., x_n)$. We want to determine the extreme points if between the variables $x_1, x_2, \ldots, x_n$ exist restrictions of the type:

$$g_i(x) = c_i,\ i = 1,..,p,\ x = (x_1, x_2, \ldots, x_n).$$

Using the Lagrange multiplicators method as follows could do the determination of extremes:

1. Building the function

$$L(x_1, x_2, ..., x_n, \lambda_1, \lambda_2, ..., \lambda_p) = f(x_1, x_2, ..., x_n) + \sum_{j=1}^{p} \lambda_j [g_j(x) - c_j]$$

2. Solving the system

$$\begin{cases} \dfrac{\partial L}{\partial x_k} = 0 \\ \ldots \\ \dfrac{\partial L}{\partial \lambda_i} = 0 \end{cases},\ k = 1,...,n;\ i = 1,...,p,$$

makes out of **n + p** equations with **n + p** unknown elements.

3. If $(x^o,\lambda^o) = (x_1^o, x_2^o,\ldots,x_n^o,\lambda_1^o,\ldots,\lambda_p^o)$

is the solution of the system, then could be calculate the matrix

$$H = \left(\frac{\partial^2 L\left(x,\lambda^0\right)}{\partial x_i \partial x_j} \right),\ i = 1,\ldots,n,\ j = 1,\ldots,n,\ \text{and then } H\big|_{x=x_0}$$

If $H\big|_{x=x_0}$ is positively defined, then x_o is a minimum point and if it is negative, then x_o is a maximum point (Cret, 2000; Intriligator, 1971).

The Geometric Study of the Method

The problem has to be studied from a geometric point of view, by using a function of production with two variables, z = z(x,y) of which graphical representation is given figure 1 (Mansfield, 1975; Mihoc, 1973)

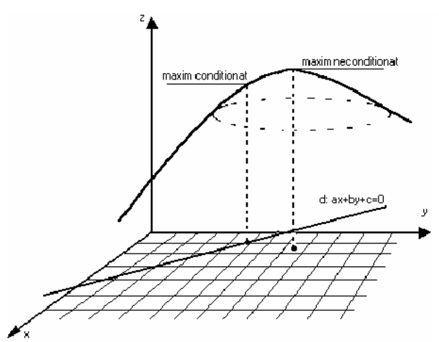

Fig. 1

If we want to determine the extremes of the function

$$z = z(x,y)$$

conditioned by the relation

$$ax + by + c = 0,$$

it must geometrically determine the point (x,y) situated on the straight line:

$$(d)\ ax + by + c = 0,$$

where z has a maximum value.

If searching in the points of the definition domain is done the determination of the extremes, it will led to the following "problems":

- If is doing a search on equal divisions (or multiple) within the unit, very few of these will be situated directly on the straight-line d, and we move off too far away from the point of conditioned extreme.
- If reducing the norm of the division (smaller than the unit) it appears a too high number of iterations, which implies a long time needed in order to solve the problem.

Therefore it proposes to search for the points of conditioned extreme on a “strip” situated along the straight-line d (figure 2).

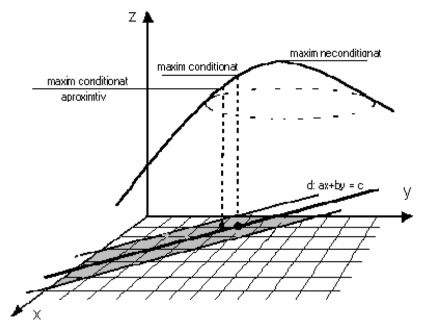

Fig. 2

The “strip” taken into consideration represents the part of the quadrant I of the plane xOy situated between the straight lines:

$$(d_1)\, ax + by = c - \alpha$$

$$(d_2)\, ax + by = c + \alpha$$

Further it presented a study regarding the dimensions of this “strip” in order to minimize the errors.

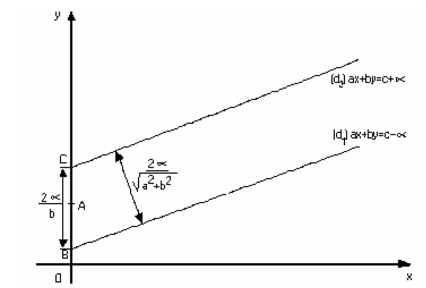

Fig. 3

Be A, B, C (figure 3) the points of intersection between the straight lines d, d_1, d_2, and Ox (Rujescu, 2005). Then:

$$A\left(0, \frac{c}{b}\right),\ B\left(0, \frac{c-\alpha}{b}\right),\ C\left(0, \frac{c+\alpha}{b}\right).$$

The distance between the two straight lines has to be calculated by choosing a point on one of them and then by calculating the distance between that point and the other straight line. Supposing that $C \in d_2$,

than

$$d(C,d_1)=\frac{\left|a \cdot 0 + b \cdot \frac{c+\alpha}{b} + (-c+\alpha)\right|}{\sqrt{a^2+b^2}}$$

$$d(C,d_1)=\frac{2|\alpha|}{\sqrt{a^2+b^2}}=\frac{2\alpha}{\sqrt{a^2+b^2}},\ \alpha > 0.$$

If the search will be done by using a division with the unit norm, any point of conditioned extreme will be inside the closed square with unit surface, determined by the four points of the netting.

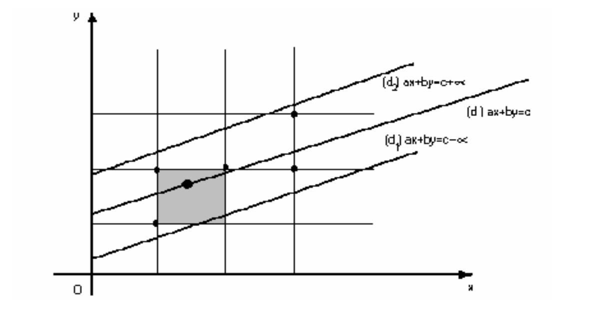

Fig. 4

The maximum distance between the point of conditioned extreme and one of the points from the netting is equal with the half of the diagonal line of the square with unit side. So the breadth of the strip will be equal with the diagonal line of the square (figure 4):

$$\frac{2\alpha}{\sqrt{a^2+b^2}}=\sqrt{2}, \quad \alpha=\frac{\sqrt{2}}{2}\sqrt{a^2+b^2}, \ \alpha=\sqrt{\frac{a^2+b^2}{2}}.$$

Conclusions

The realization of some algorithms in order to determine the conditioned extremes of the functions with more variables often asks for a deep analysis not only of the phenomenon, but also of the adopted model. A geometrical analysis of the model will point out multiple particularities.

References

Creț, F. (2000). *Elemente de modelare şi matematici speciale*. Editura Mirton, Timişoara

Intriligator, M.D. (1971). *Mathematical Optimization and Theory*. Pretince-Hall, INC, Eng., Cliffs N.J.

Mansfield, E. (1975). *Micro-Economics. Theory and applications*. W.W. Norton & Company. INC, New York

Mihoc Gh., Ştefãnescu, A. (1973). *Programarea matematică*. Editura Didactica si Pedagogica, Bucureşti

Rujescu, C. (2005). *Optimizarea mărimii exploatațiilor agricole cu ajutorul funcțiilor de producție*. Teză de doctorat, USAMVB Timisoara