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Mathematical Modeling and Cost Minimization in Functional Dairy Product Development: A Case Study on Fruit Yogurt

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Abstract
In recent years, the food industry has placed increasing emphasis on cost efficiency and nutritional quality, driven by consumer demand for affordable yet healthy products. Among dairy-based functional foods, fruit yogurt has become one of the most popular fermented products due to its balanced nutritional profile, sensory appeal, and versatility. However, developing an optimal formulation that maintains quality while minimizing production costs remains a key challenge in dairy process engineering.
This study demonstrates the application of mathematical modeling and numerical optimization to minimize the production cost of a functional fruit yogurt while maintaining essential nutritional quality standards. The optimization process was carried out using Microsoft Excel Solver, applying a linear programming approach to determine the optimal proportions of key ingredients - whole milk, skim milk powder, sugar, fruit puree, and starter culture. The objective was to minimize cost per kilogram of product, subject to constraints on protein content, fat level, and total solids. The results show that Solver can effectively identify cost-efficient formulations while ensuring compliance with quality requirements, providing a practical and educational tool for process optimization in the dairy industry.
Keywords: *Optimization, Excel Solver, Mathematical Modeling, Dairy Technology, Cost Minimization, Functional Foods, Fruit Yogurt.*

1. Introduction

Yogurt is usually produced as semisolid food products, which derived from processed animal milks under appropriate fermentation conditions with the involvement of Streptococcus salivarius ssp thermophilus and Lactobacillus delbrueckii ssp bulgaricus bacteria [1, 2]. The combination of nutritional components and live microorganisms forms the basis for a highly nutritious food that can easily complement a healthy dietary regimen [3].

The method combining linear programming theory and computer technology, affords the possibility for the Optimization problem of food prescription in engineering design. Excel's Solver function may be used to solve problems with up to 200 decision variables, enough to calculate the most problems about food prescription. Combined design requirement of prescription, the decision variables constraint condition a mathematics model of cost are established.

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The optimization prescription of minimum cost is gotten through the function of Excel programming calculation [4].
In the modern dairy industry, there is a growing demand for functional foods that combine nutritional value with affordability. Fruit yogurt represents one of the most popular functional dairy products, offering consumers a balance of probiotics, proteins, carbohydrates, and natural antioxidants from fruit ingredients. However, the cost of raw materials such as milk, fruit puree, and bacterial cultures can significantly affect product profitability. Therefore, manufacturers and researchers are increasingly applying optimization and mathematical modeling to achieve efficient formulations. The Microsoft Excel Solver is a versatile tool that applies linear and nonlinear programming to find the best possible combination of decision variables (ingredient ratios) that satisfy nutritional and economic constraints. In this study, Solver was used to minimize production cost while maintaining minimum levels of protein, fat, and total solids, ensuring the product meets both quality and sensory standards [5, 6].

2. Materials and method

According to Micula and Rotaru [7, 8] an optimization model consists of three parts as follows:
1. The variables of the model are positive real numbers to be determined. Let's have x1, x2, …, xn. There are models where these variables are positive integers, and may represent for instance the livestock in a farm.
2. The constraints to the model are m equations and inequalities containing the variables x1, x2, …, xn.
The right of each member is the resource limit restrictions addressed by restriction. This part of the mathematical model is also known as the system of restrictions.
3. The objective function contains the decision variablesx1, x2, …,xn and must be maximized/ minimized or otherwise optimized.
A system of positive values for x1, x2, …, xn which meets the restrictions is called a possible solution, and those possible solutions that optimize the objective function are called the optimal solution.
SOLVER is an Excel tool used in solving and re-optimization of linear programming problems. The method for solving linear programming is called "Simplex algorithm", which is programmable and for which there is a lot of software products (such as Matlab, Maple).
Six main ingredients were considered in the optimization model: skimmed milk, whole milk (3.5% fat), sugar, fruit puree, starter culture and stabilizer.

3. Results and discussion

For each ingredient, parameters were defined including protein content, fat content, total solids, cost per 100 g, and proportion in the final mixture (decision variable) (Table 1). The six variables in our problem are initially initialized with the value zero, and after the Solver is applied, they will take the value that leads to the optimization of the efficiency function, that is, the minimum cost.

Objective function:
Minimizing total cost per 100 g:

$C = \sum_{i=1}^{6}(c_i * x_i / 100)$,

where C represents total cost (€/100 g product),
c_i the cost per kg of ingredient i, and
x_i the fraction of ingredient i in the final mixture.

Subject to the following constraints (restriction 1)
1. $\sum_{i=1}^{6} x_i = 100$ (total mass fixed at 100 g)
2. $\sum_{i=1}^{6}(p_i*x_i/100) \geq 3\%$ (total proteins)
3. $\sum_{i=1}^{6}(f_i*x_i/100) \rightarrow 1.5\% - 3\%$
4. $\sum_{i=1}^{6}(s_i*x_i/100) \geq 12$
5. $0 \leq x_i \leq x_i,max$ and $x_i \geq x_i,min$ (ingredient limits).

The model was implemented in Microsoft Excel using SUMPRODUCT functions for total cost, protein, fat, and total solids. Solver was configured to minimize the total cost cell, with ingredient proportions as variable cells, under the defined constraints. The Simplex LP solving method was selected for linear optimization. (Table 2)

Solver was configured with the following settings:

Objective = Totals proteins cell (minimize), (formula J2)
Variable cells = Ingredient quantities, (H2:H7)
Constraints = As listed above (restriction 1)
Solving method = Simplex LP.

Table 1. Data needed to obtain minimize function for a fruit yogurt

	A	B	C	D	E	F	G	H
1	Ingredient	Protein (%) p_i	Fat (%) f_i	Total Solids (%) s_i	Cost (€/kg) c_i	Min (%)	Max (%)	Variable: Amount (%) x_i
2	Skimmed milk	3.4	0.5	9	0.8	50	80	0
3	Whole milk	15	3.5	12	1	0	20	0
4	Sugar	0	0	100	0.6	5	10	0
5	Fruit puree	0.5	0.2	30	1.2	10	15	0
6	Stabilizer	0	0	90	3	0.2	0.5	0
7	Starter culture	4	0	10	5	0.05	0.1	0

Table 2. The formulas and constraints of the problem .

I	J
Constraints	Formula_or_Value
Total cost per 100 g (J2)	=SUMPRODUCT(E2:E7,H2:H7)/100
Total mass fixed at 100 g	=SUM(H2:H7)
Total proteins (%) (J4)	=SUMPRODUCT(B2:B7,H2:H7)/100
Total fat (%) (J5)	=SUMPRODUCT(C2:C7,H2:H7)/100
Total solids (%) (J6)	=SUMPRODUCT(D2:D7,H2:H7)/100

Solver successfully identified an optimal ingredient composition that minimized total production cost while meeting all quality constraints. The optimized composition achieved the desired protein (5.01%), fat (1.0%), and total solids (20.9%) content at the lowest possible cost. These results confirm the capability of Excel Solver to provide data-driven optimization for dairy formulation.

These results can be viewed in Table 3 where the quantities of ingredients in grams were obtained (H2:H7), the cost function (formula J2) and it is also observed that the restrictions are verified (J3:J6). Thus, to make 100 grams of fruit yogurt you need: 60.4 g of skimmed milk, 19 g of whole milk, 10 g of sugar, 10 g fruit puree, 0.2 g of stabilizer and 0.05 g of starter culture and the cost function is equal to €0.87/100 g.

Table 3. The ingredients obtained in grams and the optimal total proteins, after applying the Solver

	A	H	I	J
1	Ingredient	Variable: Amount (%) x_i	Constraints	Formula_or_Value
2	Skimmed milk	60.4	Total cost per 100 g (J2)	0.87
3	Whole milk	19	Total mass fixed at 100 g	100
4	Sugar	10	Total proteins (%) (J4)	5.011
5	Fruit puree	10	Total fat (%) (J5)	1
6	Stabilizer	0.2	Total solids (%) (J6)	20.94375
7	Starter culture	0.05		

Minor adjustments in high-cost ingredients, such as fruit puree and starter cultures, led to substantial cost reductions without compromising quality. Furthermore, Solver enables dynamic re-optimization in response to fluctuating raw material prices, offering flexibility and rapid decision-making for production planning.

The current study presents a cost optimization model for the development of a functional fruit yogurt using Microsoft Excel Solver. While the results demonstrate the efficiency of linear programming for ingredient cost reduction, several important limitations must be acknowledged.

First, the model is entirely computational and theoretical. It identifies an "optimal" formulation in terms of cost and nutritional composition, but no experimental validation was performed. Physical and sensory properties—such as texture, viscosity, acidity, aroma, and stability—were not evaluated. For a fermented dairy product like yogurt, these factors are critical to consumer acceptance and product quality. Future work will therefore include laboratory-scale preparation and testing of the optimized formulation to measure pH, titratable acidity, syneresis, viscosity, and sensory attributes.

Second, the nutritional and process constraints used in the optimization model are simplified. Realistic yogurt production requires tighter control over several parameters, including starter culture concentration, stabilizer levels, sugar content for sweetness adjustment, and acceptable acidity ranges. Integrating these into a multi-constraint optimization model will make future simulations more representative of industrial practice.

Third, the cost function assumes linearity in ingredient prices. In real-world manufacturing, economies of scale, supplier discounts, and logistical costs often introduce non-linear behaviors.

Extending the current model with a piecewise-linear or non-linear cost structure—using GRG Nonlinear or Evolutionary Solver methods—would improve the model's predictive power.

Finally, the comparison with a "non-optimized base formulation" could be strengthened by clearly defining the reference recipe and sourcing its cost data from verified industrial benchmarks. Future studies will document baseline compositions and their corresponding cost structures for transparent and reproducible economic assessment.

Overall, while this study provides a robust computational framework for cost optimization, future research should integrate experimental validation, process-level parameters, and non-linear cost modeling to bridge the gap between theoretical optimization and real-world yogurt production.

4. Conclusion

Mathematical modeling is a very powerful and flexible tool that finds applicability in many fields. This paper shows how the SOLVER tools of Excel software is used easily for complex optimization problems, specifically for optimizing cost function of fruit yogurt.

The integration of mathematical modeling and optimization fosters sustainable and competitive practices in the modern dairy industry. Microsoft Excel Solver provides an effective and accessible method for cost optimization in functional food product development. The proposed linear model successfully minimized production costs while maintaining essential quality parameters (protein, fat, and total solids). Solver-based modeling supports data-driven formulation decisions, reducing reliance on empirical trial-and-error approaches. This methodology can be extended to other fermented dairy products such as kefir, flavored milk, and probiotic drinks.

Conflicts of interest / Competing interests: No conflict of interest.

Ethics approval: No ethic situation.

Availability of data and material: The data have been reviewed depending on the literature.

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