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INFLUENCE OF TEMPERATURE, FAT AND WATER CONTENT ON THE THERMAL CONDUCTIVITY OF SOME DAIRY PRODUCTS

L. Gavrilă, Adriana Fînaru, Lăcrămioara Istrati, A.I. Simion, M.E. Ciocan

University of Bacău, Research Center "Chemistry and Engineering of Natural and Synthesis Products and Environmental Protection"

Calea Mărăşeşti 157, RO-600115 Bacău, ROMANIA; lgavrila@ub.ro

Abstract

Most of the data concerning the properties of materials involved in the food processing industries are presented in tabular or graphical form. In these form, experimental data are difficult to use in calculus where properties' values are often and repeatedly necessary at different temperatures, i.e.: fluid flow, thermal balances, overall heat and/or mass transfer coefficients. This paper focuses on modelling the thermal conductivity of several products encountered in dairy technology. It aims to establish simple and reliable mathematical relationships between the up mentioned parameter and temperature, water content, or fat content. Using the literature data, different regression equations have been tested, in order to get consistent equations that express the studied parameter as a function of temperature and/or dry solids (fat) content. The obtained equations are very useful in the designing of processes and equipment for the food processing industry. They can be used in spreadsheet calculations as in other PC programs.

Keywords: *thermal conductivity, milk, cream, linear regression, dairy products*

Introduction

Unlike the other process industries (chemical, petrochemical, pulp and paper), the food industries' raw materials and products are easily degradable by external and internal factors, and most of them exhibit non-Newtonian behavior. Rationally designed equipment needs a thoroughly knowledge of physical and thermodynamical properties of materials involved in food processing: raw materials, additives, intermediates and finished products. Most of the data concerning the properties of materials involved in diary products technology are presented in tabular or graphical form: Iliescu (1971), Fox (1998), Macovei (2000). In this form, experimental data are difficult to use in calculus where property values are often and repeatedly necessary at different temperatures: fluids' flow, thermal balances, heat transfer coefficients, mass transfer coefficients.

The aim of this work is to correlate the experimental data previously published in the form of tables and/or charts, in order to obtain mathematical equations with the general form:

Property = ***f****(temperature, water content, fat content)* (1)

The obtained equations have to meet two essential requirements: simplicity (facility to use) and sufficient accuracy for engineering calculation.

Experimental

Iliescu's (1982) tabular data concerning milk thermal conductivity variation with temperature and water content and Kuk's (1955) tabular data concerning cream thermal conductivity variation with temperature and fat content were used as primary data for the regression analysis.

The data were plotted in planar coordinates ($\lambda - T$; $\lambda - W$; $\lambda - F$) and regression techniques, involving least square method were used to find the best-fit equation. Microsoft Excel™ spreadsheets, Hyams' (1996) CurveExpert®, Button's (1999) Kurv+® and Oakdale Engineering's (2003) DataFit® software were used for curves fitting and establishing of mathematical correlation equations.

Results and discussions

Thermal conductivity of whole milk (3.2% fat) varies linear with the temperature in the range of 273 – 353 K, according to equation:

$$\lambda(T) = 0.0007(T - 273) + 0.481 \qquad (1)$$

The equation (1) has a regression coefficient $R^2 = 0.9859$, and fits the experimental data of Bogdanov (1961) with a relative error of less than 0.9 %. Iliescu's (1982) equation applied to the same experimental data, gives greater relative errors, in the range of 7 – 11% (Figure 1):

$$\lambda(T) = 0.0011(T - 273) + 0.22 \qquad (2)$$

Thermal conductivity of concentrated milk (10% fat) was plotted against temperature at different values of the water content (Figure 2). Linear equations:

$$\lambda(T) = A \cdot (T - 273) + B \qquad (3)$$

were established at constant water content, parameters *A*, *B* and the correlation coefficient R^2 being presented in Table 1.

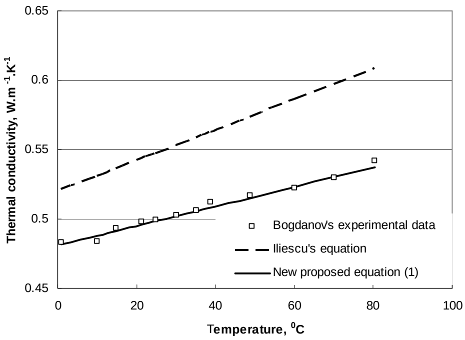

Fig. 1. Thermal conductivity of whole milk (3.2% fat) versus temperature

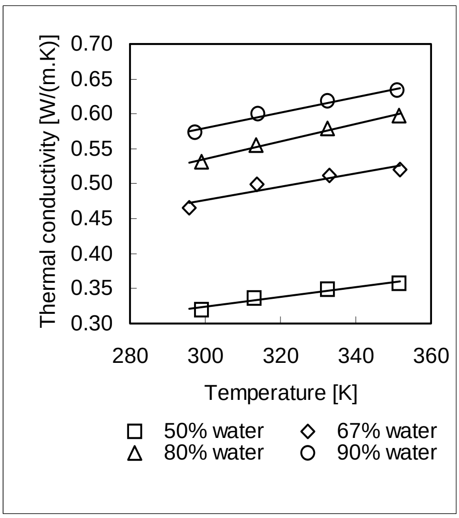

Fig. 2. Concentrated milk thermal conductivity versus temperature at different water contents

Table 1. *A* and *B* coefficients and regression coefficient R^2 from equation (3)

Water content [%]	Coefficients from equation (3)		R^2
	A	*B*	
50	0.0007	0.3042	0.9499
67	0.0009	0.4516	0.9028
80	0.0013	0.5016	0.9808
90	0.0011	0.5500	0.9772

Quadratic and logarithmic equations were also used, but the precision gained did not justify the complexity of equation. Much more, further correlations of *A* and *B* coefficients with the water content are giving more scattered values using these types of correlation.

The *A* and *B* coefficients from equation (3) have been further correlated with the water content of the concentrated milk (% weight). Due to the reduced number of experimental data, polynomial correlations were irrelevant, so the following linear equations were used:

$$A = 1 \cdot 10^{-5} W + 1 \cdot 10^{-4} \tag{4}$$

$$B = 6 \cdot 10^{-3} W + 0.0186 \tag{5}$$

Introducing equations (4) and (5) in equation (3), a two-parameter equation for thermal conductivity calculations has been established:

$$\lambda(T,W) = 1 \cdot 10^{-4}(0.1W + 1) \cdot (T - 273) + 0.006W - 0.0186 \tag{6}$$

Table 2 presents a comparison between experimental published data of Leidenfrost (1966) and conductivity values calculated with equation (6). As shown in Table 2, excepting the data for 67 % water, all other calculated values are in good concordance with experimental data, the relative error being less than 5 %. The greater the water contents of the milk, the better the correlation.

Cream thermal conductivity decreases with fat content increase, and at constant fat contents increases with the temperature. Linear correlations between cream thermal conductivity and temperature, at different fat contents are given in literature (Table 3).

Table 2. Measured and calculated values of thermal conductivity of concentrated milk with 10% fat

W [%]	*T* [K]	Thermal conductivity [W/(m.K)]		Relative error, $\|\varepsilon\|$ [%]
		literature	equation (6)	
90	297.2	0.573	0.5828	1.71
	351.1	0.634	0.6367	0.43
80	299.6	0.531	0.5225	1.59
	351.6	0.597	0.5693	4.63
67	295.8	0.466	0.4382	5.96
	351.7	0.521	0.4812	7.64
50	299.0	0.319	0.3342	4.76
	351.4	0.358	0.3656	2.13

Correlating experimental data of Kuk (1955) concerning cream thermal conductivity variation with the fat content and the temperature, the following equation has been established:

$$\lambda(T,F)=\frac{\left(-0.05F^2+3F-18\right)\cdot(T-273)-14F+3591}{10000}\left[\frac{\mathrm{W}}{\mathrm{m}\cdot\mathrm{K}}\right] \quad (13)$$

Equation (13) gives the best results, the relative error being between 0.5 – 2.75%, meanwhile equations (8 – 10) present relative errors of 3 – 17% (Figure 3, Table 4).

Table 3. Empirical equations for cream thermal conductivity (Iliescu, 1982)

Fat content, *F* [%]	λ = f(*T*)	Equation
10	0.4321 + 0.001694 × (*T*-273)	(7)
20	0.3787 + 0.001601 × (*T*-273)	(8)
35	0.3402 + 0.001531 × (*T*-273)	(9)
45	0.2485 + 0.001415 × (*T*-273)	(10)
60	0.1743 + 0.001264 × (*T*-273)	(11)
80	0.1653 + 0.000997 × (*T*-273)	(12)

Table 4. Relative errors [%] in calculating cream thermal conductivity

T [K]	*F* = 20%		*F* = 35%		*F* = 45%	
	Eq. (13)	Eq. (8)	Eq. (13)	Eq. (9)	Eq. (13)	Eq. (10)
274	1.098	12.849	0.422	8.832	1.432	17.247
282	0.544	12.639	2.232	8.583	2.740	13.498
289	1.468	11.999	0.659	4.497	2.325	13.650
294	1.745	7.375	0.880	3.144	0.974	14.658

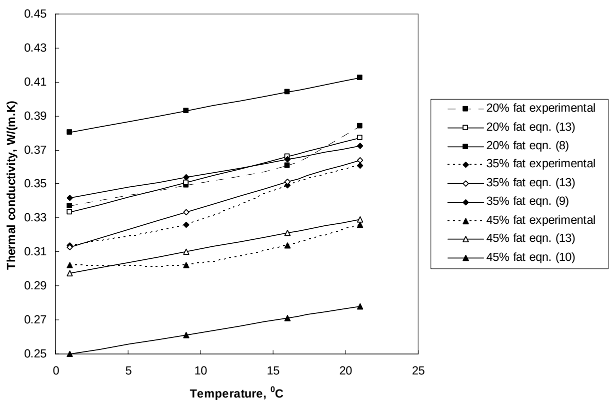

Fig. 3. Thermal conductivity of cream versus temperature, at different values of the fat content

Conclusions

Experimental data presented in literature regarding thermal conductivity of dairy products such as whole milk, concentrated milk and cream have been analysed, in order to find equations that correlate thermal conductivity with parameters such as temperature, fat content ad water (dry substance) content.

New equations have been proposed for the following correlations: thermal conductivity of whole milk with temperature, thermal conductivity of concentrated milk with temperature and water content, and thermal conductivity of cream with temperature and fat content.

The proposed equations are in better concordance with experimental data than other equations previously published in literature. The newly proposed equations are simple and easy to use in numerical calculations.

Notations

A – coefficient in equation (3);
B – coefficient in equation (3);
F – fat content, mass percent (%);
R – regression coefficient, dimensionless;
T – absolute temperature, K;
W – water content, mass percent (%);
ε – relative error, %;
λ – thermal conductivity, $W.m^{-1}.K^{-1}$;

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