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Mathematical modelling of density and viscosity of nacl aqueous solutions

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Abstract

Sodium chloride is one of the most important substances used in food industry especially as flavour enhancer and as preservative agent. It can be also involved in various other processes including drying, cooling, cryopreservation etc. The proper use of sodium chloride solutions implies a good knowledge of the behaviour of its thermophysical properties such as density, viscosity, heat capacity, entropy etc.

This paper focuses on establishing mathematical correlations between density and viscosity and parameters such as low temperature and concentration. The regression analysis leaded to several equations able to describe the cited properties. The calculated data revealed a high similarity with the existing experimental ones confirming the accuracy of the obtained mathematical models.

The developed equations can be loaded in computer spreadsheet software's for storing, organizing and manipulating data necessary for industrial and academic users and so facilitating the sizing and optimization calculations of various technological processes and industrial equipment's.

Keywords: sodium chloride, density, viscosity, thermophysical properties, mathematical modelling

1. Introduction

Sodium chloride is one of the most known and used substances. Its properties are exploited in various sectors.

In food industry, for example, it is an essential and common ingredient. Due to its ability to lower water activity, it is a very good preservative which affects the physical and sensory characteristics, shelf life and palatability [1]. For vacuum packaged meat products, the use of sodium chloride (alone or in combination with other additives) increases water holding capacity and consequently reduces cooking loss. Szerman *et al.* [2] studied the effect of cooking temperature and the incorporation of whey protein concentrate and sodium chloride on technological, physical and sensory characteristics of cooked whole-muscle beef. Their research showed that the post-injection weight loss diminished when NaCl concentration increased. Emulsified meat products such as mortadella require specific concentrations of NaCl in the original formulations to promote the extraction of myofibrillar proteins, especially complex actomyosin, which are soluble only in solutions of high ionic strength and are responsible for emulsification and fat binding properties in the batter and the formation of stable gels in the cooking stage [3].

Salt can be used also in syrup preparation for obtaining fruits products by hurdle technology - an intelligent combination of some soft treatments, or hurdles, which generates products with similar characteristics to the fresh ones, adding longer shelf

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life, high sensorial quality and guarantee microbial quality and good appearance of the products during storage. Fruits such as mango [4] or pineapple [5] were successfully processed by this method.

Also in food industry, sodium chloride is employed in the production of cereal based food. It enhances the flavour and the functional properties of cereal constituents (*e. g.* proteins and starch etc.) allowing an appropriate processing handling and final textural characteristics [6]. It plays important roles in bread production, as it influences gluten behaviour. Changes in the solvent quality due to the presence of NaCl during dough mixing result in different molecular conformation and network structure of gluten proteins which contributed to the differences in the rheological properties [7]. Sodium chloride strengthens dough structure, slows down gas production by decreasing yeast activity in the dough and enhances bread flavour. It extends the dough development time and increases its resistance, elasticity and extensibility [8].

Another important utilization of sodium chloride is based on its property to insure low temperature values. When added in water, salt requires heat to dissolve fact that reduces the solution temperature up to -20°C. Therefore it can be included in the refrigerants area. Well known as the fluids absorbing heat during evaporation, refrigerants are commonly used in refrigeration, air conditioning and heat pump systems. They can be divided in two main classes: primary refrigerants and secondary refrigerants.

The first class comprises: halocarbons, hydrocarbons, inorganic compounds azeotropic mixtures and nonazeotropic mixtures and includes four different generations. Nearly all of the developed refrigerants from the first generation were flammable, toxic, or both and some were also highly reactive. The second generation was distinguished by a shift to fluorochemicals for safety and durability. Linkage of released chlorofluorocarbons (including refrigerants) to depletion of protective ozone catalysed the third generation with focus on stratospheric ozone protection [9]. The fourth refrigerants generation is trying to reduce the use of substances known as contributing to global warming process.

The second class contains secondary refrigerants which play a role in carrying heat from an object or a space being cooled to the primary refrigerants or the evaporator of a refrigeration system. Their utilisation presents the advantage of minimizing the amounts of primary refrigerants, often known as being expensive or toxic. During this process, the secondary refrigerants have no phase change. Water-salt solutions are included in this class and even today it is still used in spite of its corrosive effects [10].

Many other uses of sodium chloride were developed since now. Salt aqueous solutions constitute a prototypical solute and solvent pair for solubility studies [11]. Dissolved sodium chloride can be used for drying on porous media [12]. Highly concentrated sodium chloride solutions can be employed to regenerate contaminant-saturated (i.e., exhausted) resins [13]. Cryopreservation processing can involve the use of salt [14].

When manufacturers from such a large variety of industrial areas need to incorporate sodium chloride into their products formulation, it is highly recommended to know if and how its thermodynamic properties will change during the processing steps. Until now various researches were realized in this direction [15-19] but the information available on density and viscosity profiles for example remain scattered.

This paper focuses on the study of the above cited thermophysical sodium chloride properties. Data provided by technical and scientific publications were used in order to establish mathematical correlations between density and viscosity and technological parameters such as low temperature and concentration.

Several equations were obtained. The calculated data revealed a high similarity with existing experimental ones confirming the accuracy of the mathematical models.

The developed equations can be loaded in computer spreadsheet software for storing, organizing and manipulating data available both for industrial and academic users and so facilitating the sizing and optimization calculations of various technological processes and equipment.

2. Material and methods

The mathematical models development and the regression analysis were based on data provided by the technical and scientific publications [20,21] (tabular and plotted data) concerning on the density and dynamic viscosity variation of aqueous sodium chloride solutions with concentration and temperature.

Different types of software were used in order to integrate the existing data and to formulate an accurate and a simple mathematical equation. The experimental data were plotted in "*Temperature – Thermo-physical property*", "*NaCl concentration in aqueous solutions – Thermo-physical property*" coordinates and different types of regression techniques were used to reveal the best-fit equation.

Because of its availability at industrial and academic level the proposed mathematical models were developed mainly in Microsoft Excel™ 2010 spreadsheets (data integration, plotting and ANOVA analysis). ANOVA analysis tool is able to offer precise and useful information in order to interpret the relations between existing data and the calculated values by the proposed mathematical models.

For more complex and atypical data plotting in 2D ("*vapour pressure*" model, "*heat capacity*" model etc.) the CurveExpert® software was employed. The representation of tabular data in 3D as a surface response was fitted and analysed in TableCurve 3D® v.4 software.

Table 1. Variation of NaCl aqueous solutions density with temperature and NaCl concentration [20]

Concentration, $C_{\%}$ [%, w/w]	Density, ρ [kg·m^{-3}]									
	Temperature, T [K]									
	273.15	283.15	293.15	298.15	303.15	313.15	323.15	333.15	353.15	373.15
1	1007.47	1007.07	1005.34	1004.09	1002.61	999.08	994.82	990.00	978.50	965.10
2	1015.09	1014.42	1012.46	1011.12	1009.57	1005.93	1001.61	996.70	985.20	971.90
4	1030.38	1029.20	1026.80	1025.30	1023.61	1019.77	1015.31	1010.30	998.80	985.50
6	1045.75	1044.08	1041.27	1039.63	1037.81	1033.78	1029.19	1024.10	1012.50	999.40
8	1061.21	1059.07	1055.89	1054.12	1052.19	1047.98	1043.26	1038.10	1026.40	1013.40
10	1076.77	1074.19	1070.68	1068.79	1066.76	1062.38	1057.53	1052.30	1040.50	1027.60
12	1092.44	1089.46	1085.66	1083.65	1081.53	1076.99	1072.02	1066.70	1054.90	1042.00
14	1108.24	1104.91	1100.85	1098.72	1096.51	1091.82	1086.74	1081.30	1069.40	1056.50
16	1124.19	1120.56	1116.21	1114.01	1111.71	1106.88	1101.7	1096.20	1084.20	1071.30
18	1140.31	1136.43	1131.90	1129.54	1127.15	1122.18	1116.91	1111.30	1099.30	1086.40
20	1156.63	1152.54	1147.79	1145.33	1142.85	1137.74	1132.38	1126.80	1114.60	1101.70
22	1173.18	1168.91	1163.95	1161.40	1158.83	1153.58	1148.12	1142.50	1130.30	1117.20
24	1189.99	1185.57	1180.40	1177.76	1175.11	1169.71	1164.14	1158.40	1146.30	1133.10
26	1207.09	1202.54	1197.17	1194.43	1191.70	1186.14	1180.45	1174.70	1162.60	1149.20

Table 2. Variation of NaCl aqueous solutions density with temperature and NaCl concentration [20]

Concentration, $C_{\%}$ [%, w/w]	Density, ρ [kg·m^{-3}]								
	Temperature, T [K]								
	258.15	263.15	268.15	273.15	283.15	288.15	293.15	313.15	333.15
10	-	-	1079.0	1078.0	1074.2	1072.6	1070.7	1062.1	1052.4
11	-	-	1087.0	1086.0	1081.8	1080.1	1078.1	1069.4	1059.5
12	-	-	10950	1093.0	1089.4	1087.6	1085.6	1076.6	1066.6
13	-	-	1102.0	1101.0	1097.1	1095.2	1093.1	1084	1073.8
14	-	--	1110.0	1108.0	1104.8	1102.8	1100.7	1091.4	1081.1
15	-	1119.0	1117.0	1116.0	1112.5	1110.5	1108.3	1098.8	1088.4
16	-	1125.0	1125.0	1124.0	1120.3	1118.2	1116.0	1106.3	1095.8
17	-	1135.0	1134.0	1133.0	1128.1	1126.0	1123.7	1113.9	1103.2
18	-	1144.0	1142.0	1141.0	1136.0	1133.8	1131.5	1121.6	1110.7
19	1151.0	1149.0	1148.0	1147.0	1143.9	1141.7	1139.4	1129.3	1118.3
20	1163.0	1162.0	1160.0	1158.0	1151.9	1149.7	1147.3	1137.1	1125.9
21	1171.0	1169.0	1168.0	1165.0	1160.0	1157.7	1155.3	1144.9	1133.6
22	1180.0	1178.0	1176.0	1174.0	1168.1	1165.7	1163.3	1152.9	1141.4
23	1187.0	1185.0	1183.0	1181.0	1176.2	1173.9	1171.4	1160.9	1149.2
24	1198.0	1196.0	1194.0	1191.0	1184.5	1182.1	1179.6	1169	1157.2
25	-	1204.0	1202.0	1199.0	1192.7	1190.1	1187.9	1177.2	1165.2
26	-	-	-	-	1201.1	1198.7	1196.3	1185.5	1173.3

Table 3. Variation of NaCl aqueous solutions dynamic viscosity with temperature and NaCl concentration [21]

Concentration, $C_{\%}$ [%, w/w]	Dynamic viscosity, μ [mPa·s]					
	Temperature, T [K]					
	293.15	283.15	273.15	268.15	263.15	258.15
0.10	10.30	12.85	17.65	-	-	-
2.90	10.39	13.24	18.04	-	-	-
5.60	10.59	13.83	18.44	-	-	-
8.30	10.98	14.42	19.12	23.05	-	-
11.0	11.47	15.20	20.20	24.42	-	-
13.6	12.26	16.18	21.48	26.09	-	-
16.2	13.14	17.26	23.24	28.34	-	-
18.8	14.32	18.53	25.59	31.18	34.91	-
21.2	15.49	20.10	28.24	34.42	38.74	47.76
23.1	16.67	21.57	30.40	37.46	43.05	52.76
24.9	18.04	23.44	32.95	40.70	47.07	57.47
26.3	19.22	25.00	35.01	-	-	-

3. Results and discussion

3.1. Density. Employing Microsoft Excel™ 2010 spreadsheets and CurveExpert® software and consideraing the best fit and the simplicity in formulation, 14 quadratic correlations have been established by plotting density ρ [kg/m^3] vs temperature T [K] at constant NaCl concentrations $C_{\%}$ [% w/w]:

$$\rho = A_1 + A_2 T + A_3 T^2 \quad (1)$$

The A_1, A_2 and A_3 values are presented in Table 4. The regression coefficients R^2 for this equations are greater than 0.999, thus indicating a good correlation of variables.

In order to correlate A_1, A_2 and A_3 coefficients with temperature $C_{\%}$ [% w/w], several models were uploaded in CurveExpert® software. The best fit model is also the quadratic equation with good regression coefficients (Table 5).

$$Coefficient = a_1 + a_2 C_{\%} + a_3 C_{\%}^{\ 2} \quad (2)$$

Combining the equations 1 and 2 the final form of proposed equation model (Equation 3) can be obtained.

Table 4. Coefficients for equations no. 1

Concentration, $C_{\%}$ [%, w/w]	Equation no. 1 coefficients			
	A_1	A_2	A_3	R^2
1	774.07326	1.80147	-0.00346	0.999
2	802.12043	1.67739	-0.00328	0.999
4	852.99849	1.46234	-0.00297	0.999
6	903.69746	1.24799	-0.00266	0.999
8	950.13882	1.06087	-0.00239	0.999
10	994.17524	0.88909	-0.00214	0.999
12	1035.55601	0.73409	-0.00192	0.999
14	1073.10274	0.60489	-0.00174	0.999
16	1109.26238	0.48429	-0.00157	0.999
18	1145.32573	0.36512	-0.00140	0.999
20	1177.58900	0.27113	-0.00127	0.999
22	1208.26220	0.18861	-0.00116	0.999
24	1241.69739	0.08996	-0.00102	0.999
26	1272.33294	0.01122	-0.00091	0.999

Table 5. Coefficients for equations no. 2

Equation 1 coefficient	Equation no. 2 coefficients			
	a_1	a_2	a_3	R^2
A_1	750.2834	26.7822	-0.26389	0.9998
A_2	1.90165	-0.11734	0.00175	0.9995
A_3	-0.003604	0.0001701	-0.00000261	0.9994

$$\rho = (a_1 + a_2 \cdot C_{\%} + a_3 \cdot C_{\%}^{\ 2})_{A_1} + (a_1 + a_2 \cdot C_{\%} + a_3 \cdot C_{\%}^{\ 2})_{A_2} \cdot T + (a_1 + a_2 \cdot C_{\%} + a_3 \cdot C_{\%}^{\ 2})_{A_3} \cdot T^2 \quad (3)$$

The calculated data with the developed mathematical model and the existing experimental data were compared (Table 6 and 7) employing the relative error equation (4).

Considering its negative and positive values an overall average of -0.00286% (0.0316% in absolute value) was obtained. The regression coefficient R^2 of the proposed model was determined as reaching a value of 0.9999.

$$\varepsilon = \left| \frac{Data_{\text{experimental}} - Data_{calculated}}{Data_{calculated}} \right| \cdot 100[\%] \quad (4)$$

The ANOVA analysis (Two-Factor with Replication) was used to compare tabular and calculated density values, at 14 different concentrations in 10 temperatures variation (Table 1). The results presented in Table 8 showed that the sample *P-value* is 0.992864, greater than the targeted alpha 0.05, and the *Fcrit* value is larger than the *F-test* value. As consequence, the null hypothesis is not rejected indicating that there is not a statistical difference between tabular and calculated data. At temperatures of 258.15, 263.15, 268.15 and 273.15 K and 16 different concentrations (Table 2) the ANOVA: Two-Factor with Replication data analysis indicate that the sample *P-value* has the value of 0.997852 (for α = 0.05) and *Fcrit* = 3.920124 is larger than the *F-test* = 7.28E-06.

Table 8. The ANOVA test summary

SUM-MARY	Temperature, *T* [K]										Total
	273.15	283.15	293.15	298.15	303.15	313.15	323.15	333.15	353.15	373.15	
Experimental data[*]											
Count	14	14	14	14	14	14	14	14	14	14	140
Sum	15428.7	15389.0	15336.4	15307.9	15277.9	15214.0	15144.2	15069.4	14903.5	14720.3	151791
Average	1102.1	1099.2	1095.5	1093.4	1091.3	1086.7	1081.7	1076.4	1064.5	1051.5	1084.2
Variance	4308.5	4127.9	3978.0	3917.1	3866.7	3785.8	3728.9	3691.8	3661.7	3666.2	3858.0
Calculated data[*]											
Count	14	14	14	14	14	14	14	14	14	14	140
Sum	15433.3	15387.0	15335.2	15307.2	15277.8	15214.8	15146.2	15072.1	14907.1	14719.8	151801
Average	1102.4	1099.1	1095.4	1093.4	1091.3	1086.8	1081.9	1076.6	1064.8	1051.4	1084.3
Variance	4287.4	4129.1	3994.8	3936.3	3883.5	3794.1	3726.1	3678.7	3644.8	3691.3	3860.5

ANOVA

Source of Variation	*SS*[**]	*df*[**]	*MS*[**]	*F*[**]	*P-value*[**]	*F crit*[**]
Sample	0.310572	1	0.310572	8.01E-05	0.992864	3.877473
Columns	65386.33	9	7265.147	1.874908	0.055938	1.915995
Interaction	1.540677	9	0.171186	4.42E-05	1	1.915995
Within	1007483	260	3874.934			

[*]*values with one decimal;* [**] *SS* - sum of squares, *df* - degrees of freedom, *MS* - mean square, *P-value* - level of significance.

Table 9. Coefficients for equation no. 5

Coefficient	Value	Coefficient	Value
b_1	919.0202567	b_4	0.027175948
b_2	8.661163416	b_5	-0.00199299
b_3	0.854264859	b_6	-0.00585389

Table 6. Calculated densities values with the proposed mathematical model (Equation no. 3) and the relative errors between predicted values and experimental data (Table 1)

Conc. $C_{\%}$ [%, w/w]	Temperature, T [K]																			
	273.15		283.15		293.15		298.15		303.15		313.15		323.15		333.15		353.15		373.15	
	ε, %*	PV**	ε, %	PV	ε, %	PV	ε, %	PV	ε, %	PV	ε, %	PV	ε, %	PV	ε, %	PV	ε, %	PV	ε, %	PV
1	0.08	1008.3	0.01	1007.0	0.03	1005.1	0.03	1003.8	0.02	1002.4	0.00	999.1	0.03	995.1	0.04	990.4	0.05	979.0	0.03	964.8
2	0.06	1015.7	0.01	1014.3	0.03	1012.1	0.03	1010.8	0.02	1009.4	0.00	1005.9	0.02	1001.8	0.04	997.1	0.04	985.6	0.04	971.5
4	0.04	1030.8	0.03	1028.9	0.04	1026.4	0.03	1025.0	0.02	1023.4	0.01	1019.7	0.01	1015.4	0.03	1010.6	0.03	999.1	0.03	985.2
6	0.03	1046.1	0.03	1043.8	0.03	1040.9	0.03	1039.3	0.02	1037.6	0.01	1033.7	0.01	1029.3	0.02	1024.3	0.03	1012.8	0.03	999.1
8	0.02	1061.4	0.03	1058.8	0.02	1055.6	0.02	1053.9	0.02	1052.0	0.01	1047.9	0.01	1043.3	0.01	1038.3	0.03	1026.7	0.02	1013.2
10	0.02	1077.0	0.02	1074.0	0.01	1070.5	0.01	1068.7	0.01	1066.7	0.00	1062.4	0.01	1057.6	0.01	1052.5	0.03	1040.8	0.01	1027.4
12	0.02	1092.7	0.01	1089.3	0.00	1085.6	0.00	1083.6	0.00	1081.5	0.00	1077.0	0.01	1072.2	0.02	1066.9	0.03	1055.2	0.01	1041.9
14	0.03	1108.5	0.00	1104.9	0.01	1100.9	0.01	1098.8	0.01	1096.6	0.01	1091.9	0.02	1086.9	0.02	1081.6	0.04	1069.8	0.01	1056.6
16	0.03	1124.5	0.01	1120.6	0.02	1116.4	0.02	1114.2	0.02	1111.9	0.02	1107.1	0.02	1101.9	0.02	1096.5	0.04	1084.6	0.02	1071.5
18	0.03	1140.7	0.01	1136.5	0.02	1132.1	0.02	1129.8	0.02	1127.4	0.02	1122.4	0.02	1117.1	0.03	1111.6	0.04	1099.7	0.02	1086.7
20	0.03	1157.0	0.01	1152.6	0.02	1148.0	0.02	1145.6	0.02	1143.1	0.02	1138.0	0.02	1132.6	0.02	1127.0	0.03	1115.0	0.02	1102.0
22	0.03	1173.5	0.00	1168.9	0.01	1164.1	0.02	1161.6	0.02	1159.1	0.02	1153.8	0.02	1148.3	0.01	1142.6	0.02	1130.5	0.03	1117.5
24	0.01	1190.1	0.02	1185.4	0.00	1180.4	0.00	1177.8	0.01	1175.2	0.01	1169.8	0.01	1164.2	0.00	1158.4	0.00	1146.2	0.01	1133.2
26	0.02	1206.9	0.05	1202.0	0.03	1196.9	0.02	1194.2	0.01	1191.6	0.01	1186.1	0.00	1180.4	0.01	1174.5	0.03	1162.2	0.00	1149.2

*ε, % - Relative error;
**PV - Predicted Value.

Table 7. Calculated densities values with the proposed mathematical model (Equation no. 3) and the relative errors between predicted values and experimental data (Table 2)

Concentration, $C_{\%}$ [%, w/w]	Temperature, T [K]							
	258.15		263.15		268.15		273.15	
	ε, %*	PV**	ε, %	PV	ε, %	PV	ε, %	PV
10	-	-	-	-	0.06	1079.0	0.09	1078.0
11	-	-	-	-	0.07	1087.0	0.11	1086.0
12	-	-	-	-	0.07	10950	0.03	1093.0
13	-	-	-	-	0.02	1102.0	0.04	1101.0
14	-	-	--	--	0.02	1110.0	0.05	1108.0
15	-	-	0.09	1119.0	0.11	1117.0	0.05	1116.0
16	-	-	0.28	1125.0	0.12	1125.0	0.05	1124.0
17	-	-	0.12	1135.0	0.04	1134.0	0.04	1133.0
18	-	-	0.05	1144.0	0.06	1142.0	0.03	1141.0
19	0.33	1151.0	0.34	1149.0	0.25	1148.0	0.16	1147.0
20	0.01	1163.0	0.07	1162.0	0.08	1160.0	0.08	1158.0
21	0.04	1171.0	0.04	1169.0	0.05	1168.0	0.02	1165.0
22	0.00	1180.0	0.01	1178.0	0.03	1176.0	0.04	1174.0
23	0.12	1187.0	0.11	1185.0	0.09	1183.0	0.07	1181.0
24	0.09	1198.0	0.11	1196.0	0.13	1194.0	0.07	1191.0
25	-	-	0.07	1204.0	0.10	1202.0	0.04	1199.0

*ε, % - Relative error; **PV - Predicted Value.*

By plotting directly the tabular data for the density in TableCurve 3D® v.4 software, an equation for the response surface was generated. The Equation no. 5 is a polynomial equation, Rank 106, Eqn. 301 in TableCurve 3D® v.4 library with a precision of R^2 = 0.99984445, *FitSdErr* = 0.78898855, *Fstat.* = 172266.37. Its coefficients are presented in Table 9.

$$\rho = b_1 + b_2 \cdot C_{\%} + b_3 \cdot T + b_4 \cdot C_{\%}^2 + b_5 \cdot T_{\%}^2 + b_6 \cdot C_{\%} \cdot T \quad (5)$$

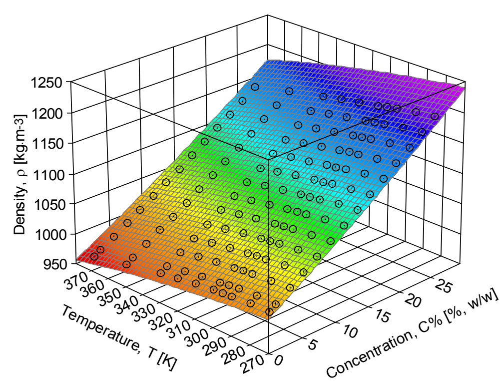

Figure 1. NaCl aqueous solutions density values plotted in TableCurve 3D and fitted with polynomial type (Equation no. 5) and its residuals

3.2. Dynamic viscosity. With the help of Microsoft Excel™ 2010 spreadsheets and CurveExpert® software, 12 linear correlations (equation 6) have been established by plotting the logarithmic values of dynamic viscosity μ [mPa·s] vs temperature T [K] at constant NaCl concentrations $C_{\%}$ [% w/w].

$$\log(\mu) = A_1 + A_2 T \quad (6)$$

The A_1 and A_2 values are presented in Table 10.

In order to correlate A_1 and A_2 coefficients with the NaCl concentrations $C_{\%}$ [% w/w], several models were generated in available software (1st, 2nd and 3rd degree polynomial equations, "vapour pressure" model, "heat capacity" model etc.). The best fit model is the quadratic equation 7 (Table 11).

$$Coefficient = a_1 + a_2 C_{\%} + a_3 C_{\%}^2 \quad (7)$$

Combining the equations 6 and 7 and, the final form of the proposed model (Equation 8) is:

$$\log(\mu) = (a_1 + a_2 \cdot C_{\%} + a_3 \cdot C_{\%}^2)_{A_1} + (a_1 + a_2 \cdot C_{\%} + a_3 \cdot C_{\%}^2)_{A_2} \cdot T \quad (8)$$

The calculated value with the developed mathematical model and the existing tabular value were compared (Table 12). An overall average error of -0.46% (2.11% in absolute value) was calculated. The regression coefficient R^2 determined for the proposed model was 0.971.

If the temperature is measured in Celsius degrees the equation no. 8 becomes:

$$\log(\mu) = (a_1 + a_2 \cdot C_{\%} + a_3 \cdot C_{\%}^2)_{A_1} + (a_1 + a_2 \cdot C_{\%} + a_3 \cdot C_{\%}^2)_{A_2} \cdot t + (a_1 + a_2 \cdot C_{\%} + a_3 \cdot C_{\%}^2)_{A_3} \cdot t^2 \quad (9)$$

Its coefficients are presented in Table 12.

The calculated value with the Equation no. 9 and the existing tabular value were compared (Table 13) employing the same relative error equation (4) with an overall average of -0.18% (1.31% in absolute value). The regression coefficient R^2 in this case was determined as 0.986.

Table 10. Coefficients for equations no. 6

Concentration, $C_{\%}$ [%, w/w]	Equation no. 6 coefficients A_1	A_2	R^2
0.10	4.4344	0.0117	0.989
2.90	4.5240	0.0120	0.995
5.60	4.5539	0.0120	0.999
8.30	4.7607	0.0127	0.996
11.0	4.8476	0.0129	0.997
13.6	4.8682	0.0129	0.996
16.2	4.9751	0.0132	0.995
18.8	5.0434	0.0133	0.995
21.2	5.2881	0.0140	0.995
23.1	5.4361	0.0144	0.995
24.9	5.4973	0.0145	0.995
26.3	5.0958	0.0130	0.995

Table 11. Coefficients for equations no. 7

Equation no. 6 coefficients	Equation no. 7 coefficients a_1	a_2	a_3	R^2
A_1	4.4520193493	0.0207516148	0.0008479277	0.9998
A_2	0.0117547070	0.0000662374	0.0000017647	0.9995

Table 12. Coefficients for equations no. 9

Coefficient	Equation no. 9 coefficients a_1	a_2	a_3
A_1	1.2468403652	0.0016879755	0.0003714418
A_2	0.0152981340	- 0.0002004543	0.0000076424
A_3	0.0001564655	- 0.0000108080	0.0000003203

Table 13. Calculated dynamic viscosity values with the proposed mathematical model (Equation no. 8) and the relative errors between predicted values and experimental data (Table 3)

Concentration, $C_{\%}$ [%, w/w]	Dynamic viscosity, μ [mPa·s]											
	Temperature, T [K]											
	293.15		283.15		273.15		268.15		263.15		258.15	
	ε, %[*]	PV[**]	ε, %	PV	ε, %	PV	ε, %	PV	ε, %	PV	ε, %	PV
0.10	1.50	10.15	3.51	13.30	1.20	17.44	-	-	-	-	-	-
2.90	0.88	10.30	2.45	13.56	0.97	17.87	-	-	-	-	-	-
5.60	0.21	10.57	1.15	13.99	0.42	18.52	-	-	-	-	-	-
8.30	0.14	10.96	1.22	14.60	1.63	19.43	2.74	22.42	-	-	-	-
11.0	0.29	11.50	1.37	15.41	2.19	20.64	2.17	23.89	-	-	-	-
13.6	0.70	12.17	1.45	16.41	3.04	22.13	1.49	25.70	-	-	-	-
16.2	0.94	13.02	2.42	17.68	3.30	24.01	1.29	27.98	-	-	-	-
18.8	1.80	14.06	3.85	19.24	2.91	26.33	1.20	30.81	3.23	36.04	-	-
21.2	1.61	15.24	4.55	21.01	2.61	28.98	1.15	34.03	3.13	39.95	1.77	46.92
23.1	1.96	16.34	5.14	22.68	3.52	31.47	1.04	37.07	1.43	43.67	2.51	51.44
24.9	2.71	17.55	4.56	24.51	3.86	34.22	0.65	40.44	1.51	47.78	1.75	56.46
26.3	3.14	18.62	4.50	26.13	4.72	36.66	-	-	-	-	-	-

[*]*ε, % - Relative error;*
[**]*PV - Predicted Value.*

The ANOVA: Two-Factor with Replication analysis was used to compare tabular and calculated dynamic viscosity values (Table 3). The results presented showed that the sample *P-value* is 0.95941 greater than the targeted alpha 0.05 and the *Fcrit* value of 3.912875 is larger than the *F-test* value of 0.002600079 indicating that there is not a statistical difference between tabular and calculated data.

By plotting the tabular data for the dynamic viscosity in TableCurve 3D® v.4 software multiple equations were generated, but only one equation was selected taking into consideration its high precision. The Equation no. 10 is a linear equation, Rank 39, Eqn. 151733687 in TableCurve 3D® v.4 library with R^2 = 0.9980648444, *FitSdErr* = 0.9978009596, *Fstat.* = 0.5441760631. Its coefficients are presented in Table 14.

$$\ln(\mu) = b_1 + b_2 \cdot C_{\%}^2 \cdot \ln(C_{\%}) + b_3 \cdot e^{\frac{C_{\%}}{wC_{\%}}} + b_4 \cdot \ln(T) + \frac{b_5}{T^{0.5}} + b_6 \cdot \frac{\ln(T)}{T} \quad (10)$$

Table 14. Coefficients for equation no. 10

Coefficient	Value	Coefficient	Value
b_1	-18335.9314	b_4	2040.636458
b_2	0.000204573	b_5	185548.199
b_3	0.25824408	b_6	-211131.092
$wC_{\%}$	42.0001357		

3.3. Kinematic viscosity. In order to calculate the kinematic viscosity (ν) of NaCl aqueous solutions it is necessary to combine the developed mathematical models for dynamic viscosity and density according using equation 11:

$$\nu = \frac{\mu}{\rho} \ [m^2 \cdot s^{-1}] \quad (11)$$

4. Conclusion

The developed mathematical model accuracy for the density of NaCl aqueous solution in range of 258.15-373.15 K temperature and of 1-26% w/w NaCl concentration is sustained by the low value of absolute relative error (0.0316%), ANOVA analysis and the regression coefficient of 0.9999. In the case of dynamic viscosity model (258.15-293.15 K and 0.1-26.3% w/w NaCl concentration) the value registered for the $\varepsilon\%$ was 2.11% and R^2 was 0.971 for temperature measured in Kelvins and $\varepsilon\%$ was 1.31% and R^2 was 0.986 for temperature measured in Celsius degrees.

The proposed mathematical models for density and viscosity can be used as tools that could be loaded independently or in association in various PC software's, allowing for the targeted concentrations and temperatures, the values of the studied thermophysical properties to be found easier and with a higher precision than using the existing experimental data in tabular form.

These mathematical models were developed for software's used in storing, organizing and manipulating data available both for industrial and academic users and so facilitating the sizing and optimization calculations of various technological equipment and processes.

Compliance with Ethics Requirements. Authors declare that they respect the journal's ethics requirements. Authors declare that they have no conflict of interest and all procedures involving human / or animal subjects (if exist) respect the specific regulation and standards.

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